Graded Universal Function for BSS-Grzegorczyk Hierarchy and Separation of Complexity Classes

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In this communication we will assume some basic knowledge of Blum-Shub and Smale model of computation (see [1]). We will first define an extension of Grzegorczyk hierarchy (see [3] for the definition of this classical hierarchy) to the framework of BSS-model of computation (extending ideas from [2]). We show that this BSS-Grzegorczyk hierarchy is strict. As in the classical case a hierarchy of classes of relations is associated to BSS-Grzegorczyk hierarchy. By building a graded universal function for this hierarchy we prove that it is a strict hierarchy from the third level. The method of proof (which has some reminiscence from [4]) gives us a unified proof of the classical result. But it has no consequence on the main open problem in the subject : separation of the three first levels of this hierarchy of relations.

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[4] Ritchie R. W., Classes of predictably computable functions, Trans. Amer. Math. Soc. 106 (1963) 139–179.

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