Is 'Analytical Truth' in Mathematics Co-Extensional with 'Theorem?

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Here, a theorem is a sentence which is yielded by an effective proof procedure, 'analytically true' can mean *a)* true in virtue of meaning, *b)* tautologous, or *c)* follows from axioms and definitions alone.

An argument for the thesis that 'analytically true', when applied to the mathematical context, is co-extensional with 'theorem' is as follows, 'Analytic' is an epistemological term. There is a positive and a negative characterisation, both of which can be found in Frege. The positive characterisation is *c* above, The negative characterisation is: a statement is analytically true just in case to know that it is true we neither have to appeal to sense perception, nor do we need to appeal to Kantian spatial or temporal intuition. Looking at the notation of the Begrifsschrift and the Grundgesetze together with the notion of a gapless proof, we have a good idea of how to interpret 'follows from' in *c*. In hindsight, we can replace 'follows from' with 'yielded by an effective, proof procedure'. Under the *c* understanding of analyticity in mathematics and the 'effective proof understanding of 'follows from' we have a position which holds that 'analytically true' in mathematics is co-extensional with 'theorem',

In contrast, an argument against the thesis runs as follows. Under definitions *a* and *b*, 'analytic' is a semantic notion. Apprised of the Gö del incompleteness theorem we know that truth in a sufficiently sophisticated system is not co-extensional with theorem, of that system. We can now jump one of two ways. *1)* we can seek to preserve the thesis and then conclude that 'analytic truth' in mathematics is restricted to complete systems, or significant proportions of incomplete system; or *2)* we can drop the thesis. *1* is not an interesting option, for it makes the thesis empty or trivial,

To argue for *2* we consider the Godel sentences, argue that they are analytically true, and then draw out the significance of the qualification of 'analytic' to a sentence in mathematics. I argue that the significance is tied to the philosophical notion of an ultimate justification.

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