Unfolding Finitist Arithmetic

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The notion of *unfolding a schematic formal system* was introduced in Feferman [1] in order to answer the following question:

> *Given a schematic system* S*, which operations and predicates, and which principles concerning them, ought to be accepted if one has accepted* S*?*

A paradigmatic example of a schematic system S is the basic system NFA of non-finitist arithmetic. In Feferman and Strahm [2], three unfolding systems for NFA of increasing strength have been analyzed and characterized in more familiar proof-theoretic terms; in particular, it was shown that the full unfolding of NFA, $\mathcal{U}(\mathsf{NFA})$, is proof-theoretically equivalent to predicative analysis.

The aim of the present contribution is to elucidate the concept of unfolding in the context of the schematic system for *finitist arithmetic* FA in order to answer the crucial question of what principles are implicit in the actual finitist conception of the set of natural numbers. The main results state that the operational unfolding $\mathcal{U}_0(\mathsf{FA})$ and even the full (predicate) unfolding $\mathcal{U}(\mathsf{FA})$ of finitist arithmetic FA conservatively extend Skolem's system of primitive recursive arithmetic PRA. These results support Tait's informal analysis of finitism in [3] according to which finitist mathematics does not go beyond PRA.

[1] Feferman, S. Gödel's program for new axioms: Why, where, how and what? In *Gödel '96*, P. Hájek, Ed., vol. 6 of *Lecture Notes in Logic*. Springer, Berlin, 1996, pp. 3–22.

[2] Feferman, S., and Strahm, T. The unfolding of non-finitist arithmetic. *Annals of Pure and Applied Logic*. To appear.

[3] Tait, W. Finitism. *Journal of Philosphy 78* (1981), 524–546.

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