Continuous Liftings

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Conjecture: Assuming a sufficiently strong forcing axiom (e.g., PFA), for every pair I, J of projective ideals on N, every isomorphism between their quotient algebras has a continuous lifting. (An ideal is projective if it corresponds to a projective subset of the Cantor space.)

An affirmative answer to the above would imply that two quotient algebras are isomorphic if and only if there is a definable isomorphism. This was confirmed for P-ideals in [1], using an axiom much weaker than PFA. I will survey recent progress on the question, and explain connections with the structure of gap-spectra in quotient algebras.

The above should not be confused with a complementary conjecture of Todorcevic, that every homomorphism between quotients over analytic P-ideals with a continuous lifting has a completely additive lifting (see [1]). While the latter is concerned with the structure of definable homomorphisms, the former is concerned with the interplay between definable and arbitrary sets of reals. It belongs to a program of investigating when are the connecting maps between definable objects necessarily definable.

References.

[1] I. Farah, Analytic quotients: theory of liftings for analytic ideals over the integers, Memoirs of the AMS, to appear.

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