On the Generalized Positive Comprehension Principle

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In this communication, we present the theory GPK^+_∞ (due to M. Forti, R. Hinnion, F. Honsell, E. Weydert, M. Boffa). This theory has a comprehension scheme for bounded positive formulas and an axiom scheme of *closure*: for any *class* A of sets (i.e. any definable collection of sets), there is a smallest *set* $\overline{A}$ including A.

We can show that this closure operation behaves like a *topological closure*. It is thus natural to call a class A *closed* iff $\overline{A} = A$, which is equivalent to say that A is a set. Notice that this theory has a universal set $V = \{x \mid x = x\}$ ("$x = x$" is positive).

Our main result about this theory is that it is mutually interpretable with KM + "*On* has the tree-property"; KM is the Kelley-Morse class-theory; "*On* has the tree-proprety" is the natural translation to the class of ordinals of the corresponding notion for cardinals in ZF. The sets of the interpretation of KM in GPK^+_∞ are the well-founded sets of GPK^+_∞ and the classes are the classes (in the sense of GPK^+_∞) of well-founded sets; these classes can be represented in GPK^+_∞ by their closure using the fact that the well-founded sets are *isolated* in the topological sense. The proof of this result is among other inspired from results on *hyperuniverses*, the "natural" models of GPK^+_∞ (see f.e. [3]).

Another interesting result on GPK^+_∞ is that the axiom of choice is inconsistent with it ([2]).

[1] Esser O. *On the Consistency of a Positive Theory.* Mathematical Logic Quarterly **45** (1999).

[2] Esser O. *Inconsistency of the axiom of choice with* GPK^+_∞. To appear in J. Symb. Logic.

[3] Forti M. and Hinnion R. *The Consistency Problem for Positive Comprehension Principles.* J. Symb. Logic **54** (1989), pp. 1401–1418.

(For a complete list of references, see [2])

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