On Wonderful Extensions of the Field Q of Rational Numbers

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A field F of characteristic 0 is called a wonderful extension of the field Q if there are the embeddings (l(r);l(p),p in P) of the field F into the field R of real numbers and the fields Qp of p-adic numbers for p in P (here P is the set of all prime numbers) such that the following three conditions are satisfied:

1.Condition NB: For any element a in F for almost all p in P l(p)(a) lies in the ring of p-adic integers.

2.Condition M: There is no proper algebraic extension of the field F having embeddings into all fields Q_p.

3.Hasse's principle for smooth rational points of algebraic varieties defined over F relative to embeddings l(r);l(p),p in P.

Theorem 1 (Existence) Any countable field F of characteristic 0 with embeddings l(r);l(p),p in P,satisfing condition NB,has a countable wonderful extension.

Theorem 2 ('Uniqueness') a) Any two wonderful extensions are elementary equivalent. b) Any extension of wonderful extensions is elementary.

Theorem 3 (Decidability) The elementary theory of the wonderful extensions is decidable.

The proofs of theorem 1 and 2 are based on the model theory of multi-valued fields with near boolean (NB) families of valuation rings developed in [1].For the proof of theorem 3 it needs the details of the elementary theory of prime finite fields from [2].

[1] Yu.L.Ershov, Multi Valued Fields, Scientific Books, Novosibirsk,2000

[2] M.Fried, M.Jarden, Field Arithmetic, Springer-Verlag, 1986

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