Real and Surreal Arithmetic Continua

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In his monograph On Numbers and Games [1], J. H. Conway introduced an ordered field No which contains (in a suitable sense that can be made precise) "all numbers great and small." However, in addition to its distinguished structure as an ordered field No has a rich algebraico-tree-theoretic structure, or simplicity hierarchy, that emerges from the recursive clauses in terms of which it is defined. In a recent paper [2], we introduced a class of ordered fields with simplicity hierarchies whose properties generalize those of No and drew attention to some of the important relations that exist between No and this more general class of s-hierarchical ordered fields as we call them. Among a host of other results, we showed that an ordered field together with a tree structure is an s-hierarchical ordered field if and only if it is isomorphic to an ordered field that is an initial subtree of No (a property that is satisfied by every real-closed ordered field). In the present paper, we examine the relation between No and its unique s-hierarchical ordered subfield of reals and establish a variety of theorems which naturally suggest that whereas the s-hierarchical ordered field of reals should merely be regarded as constituting an Archimedean s-hierarchical arithmetic continuum, No may be regarded as a sort of absolute s-hierarchical arithmetic continuum (modulo von Neumann-Bernays-Gödel set theory). For proofs of these and related results, see [3]

[1] J.H. Conway, On Numbers and Games, Academic Press, 1976.

[2] P. Ehrlich, Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers, The Bulletin of Symbolic Logic, vol. 4, no. 2 (1998), p. 212.

[3] P. Ehrlich, Number systems with simplicity hierarchies: A generalization of Conway's theory of surreal numbers, The Journal of Symbolic Logic (forthcoming).

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