O-minimal Cohomology and Definably Compact Definable Groups

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Let N an $\aleph_1$-saturated o-minimal expansion of a real closed field. We develop the theory of covers for N-spaces and the cohomology theory for the category of closed and bounded N-definable sets with continuous N-definable maps and use this to solve the problem of existence of torsion points on N-definably compact N-definable abelian groups. Namely we prove the following result: Let G be an N-definably compact N-definably connected N-definable group. Then the o-minimal Euler characteristic of G is zero. Moreover, if G is abelian and $\pi_1(G) = \mathbb{Z}^n \oplus \mathbb{Z}_{m_1^{l_1}} \oplus \cdots \oplus \mathbb{Z}_{m_r^{l_r}}$, then $n > 0$ and for all $k > 1$, $card(\{x \in G : kx = 0\}) = k^n \cdot (k, m_1) \cdots (k, m_r)$.

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