Recursive Upper Bounds for a Class of L^n-Theories

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M. Grohe has shown (see [1]) that if $n \geq 3$ and the vocabulary of L contains at least one $(n-1)$-ary relation symbol then there is *no* recursive function f such that for all L^n-theories T with finite models, $\min\{|M| : M \models T\} \leq f(|S_n^n(T)|)$, where $S_n^n(T)$ is the set of L^n-types in n free variables with respect to T. In [2], for every $n \geq 2$, a well behaved class of L^n-theories (where the vocabulary of L may contain relation symbols of all arities $\leq n$), call it $\mathcal{T}_n$, is studied. There it is proved that every theory in $\mathcal{T}_n$ has finite models, and in this talk we show that there exists a recursive function $f : \omega^2 \to \omega$ such that for every $n \geq 2$ and every $T \in \mathcal{T}_n$, $\min\{|M| : M \models T\} \leq f(n, |S_n^n(T)|)$. This result depends heavily on the theory of stable homogeneous structures, a survey of which can be found in [3].

[1] M. Grohe, *Large finite structures with few L^k-types*, ***Proceedings of the 23rd International Symposium on Mathematical Foundations of Computer Science***, Lecture Notes in Computer Science 1450, Springer-Verlag, (1998).

[2] M. Djordjević, *Finite variable logic, stability and finite models*, to appear in ***J. Symb. Logic***.

[3] A. H. Lachlan, *Stable finitely homogeneous structures, a survey*, in: B. T. Hart et al. (Eds.) ***Algebraic Model Theory***, Kluwer Academic Publishers (1997), 145-159.

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