Parametrized Partitions

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A. Miller [1] showed that for every Borel $f : \omega^\omega \times [\omega]^\omega \to 2$, there is a perfect set $P \subseteq \omega^\omega$ and an infinite $H \subseteq \omega$ such that f is constant on the product $P \times [H]^\omega$. This can be seen as a Galvin-Prikry parametrization of the Berstein property. Parametrized partition properties of this type have interesting applications.

Infinite products of finite sets of natural numbers constitute particularly nice perfect subsets of the Baire space, and it is interesting to consider the possibility of parametrizing the corresponding strengthening of the Bernstein property. We show that such a parametrization is possible, even for analytic partitions. Moreover, we show that for every sequence $\{m_i\}$ of natural numbers there is an algebra $\mathbb{PCL}$ of subsets of $\omega^\omega \times [\omega]^\omega$ such that for every $\mathbb{PCL}$-measurable function $f : \omega^\omega \times [\omega]^\omega \to 2$, there exists a sequence $\{H_i : i \in \omega\}$ with $H_i \subseteq \omega$ and $|H_i| = m_i$ for every $i \in \omega$, and a set $H \in [\omega]^\omega$, such that f is constant on $\prod_{i\in\omega} H_i \times [H]^\omega$. The algebra $\mathbb{PCL}$ contains the Borel subsets of $\omega^\omega \times [\omega]^\omega$ and it is closed under Suslin's operation. Moreover, it contains all subsets of $\omega^\omega \times [\omega]^\omega$ in $L(\mathbb{R})$ when $L(\mathbb{R})$ is a Solovay model. This can be used to answer a question of [2] regarding the relationship between a strong infinite dimensional polarized partition relation and $\omega \to (\omega)^\omega$.

[1] Miller, A. W., Infinite combinatorics and definability, *Annals of Pure and Applied Logic* 41 (1989), 179-203.

[2] Di Prisco, C. A. and J. Henle, Partitions of Products, *Journal of Symbolic Logic* 58 (1993), 860-871.

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