Minimal Groups in Separably Closed Fields

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This is joint work with Elisabeth Bouscaren.

Let us recall the classical situation:
Over an algebraically closed field any connected 1-dimensional algebraic group is isomorphic to either the additive or multiplicative group or to an elliptic curve. Any definable group in an algebraically closed field K is definably isomorphic to the group of K-rational points of an algebraic group defined over K and dimension and Morley rank coincide.

Separably closed not algebraically closed fields are stable not superstable, and furthermore no definable infinite set is ranked. Hence we have to work with infinitely definable objects in order to get potentially ranked objects. An example is $K^{p^\infty} := \cap_{n\in\mathbb{N}} K^{p^n}$, which is a (pure algebraically closed field hence a) rank 1 subfield of K. Suppose now that G is a minimal group infinitely definable in K separably closed of finite imperfection degree. This means that G is U-rank 1 and connected (in the sense of stability), hence abelian and either divisible or of torsion p. We prove that, in the first case, G is definably isomorphic either to the multiplicative group of K^{p^∞} or, for a simple abelian variety A defined over K, to the divisible hull of the group $A(K)$, both groups having finite transcendence degree, *i.e.* a "thin" generic type. In the second case G is definably isomorphic to a λ-closed subgroup of the additive group and may have infinite transcendence degree.

A crucial fact is that any group definable in K is definably isomorphic to the subgroup of K-rational points of some algebraic group defined over K.

Groups definable in separably closed fields, Bouscaren and Delon.

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