The Many Senses of Completeness

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A century ago, as well as today, the notion of completeness had many different and not always equivalent senses. Both Edmund Husserl and David Hilbert introduced more than one notion of completeness in the last years of the 19^{th} Century. These concepts, albeit differently motivated, have nonetheless certain similarities. Husserl himself thought that the notion of completeness related to Hilbert's axiom of completeness derived from the concepts of completeness that he introduced in order to deal with the problem of imaginary entities in mathematics.

In this paper, which I conceive as a sequel to my "Husserl's two notions of completeness"(to appear in *Synthese*), I want to consider this question in more details than has been done before in the literature. Beginning with a careful analysis of the notions of completeness defined in paragraph 72 of *Ideen I*, I try to put Husserl's ideas concerning formal systems and their properties into the broader context of his philosophy. In particular his ideas concerning formal ontology as a province of formal logic.

The fact that the study of formal systems belonged, according to Husserl, to formal ontology and not, as Hilbert thought, to a purely combinatorial metamathematics, explains, I believe, why Husserl thought that the property of deductive completeness, which Hilbert considered as purely syntactic, is related to the property of semantic completeness implicit in the axiom of completeness that Hilbert introduced in the axiomatisation of geometry and the theory of real numbers.

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