Invariants of Semi-algebraic p-adic Sets

R. Cluckers

(Joint work with D. Haskell, Department of mathematics, College of the Holy Cross, Worcester, Ma, USA).

We consider p-adic semi-algebraic sets, that is, definable subsets of $\mathbb{Q}_p^k$ in the language of fields with n-th power predicates in which we have quantifier elimination [M]. We prove via an explicit construction that there is a definable bijection between $\mathbb{Z}_p$ and $\mathbb{Z}_p \setminus \{0\}$. It follows that there is no non-trivial Euler characteristic on $\mathbb{Q}_p$. This result can analogously be obtained for a large class of Henselian valued fields with appropriate language.
More generally, we use the cell decomposition [D] to prove that there is a definable bijection between two definable sets if and only if they have the same dimension (as defined by [SD]). The proof works also for finite field extensions of $\mathbb{Q}_p$.

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[SD] Scowcroft, P.; van den Dries, L.: *On the structure of semialgebraic sets over p-adic fields* J. Symbolic Logic 53 (1988), no. 4, 1138-1164.

Address: Department of Mathematics, University of Leuven, Celestijnenlaan 200B, 3001 Leuven, BELGIUM
E-mail: raf.cluckers@wis.kuleuven.ac.be