Some Asymptotic Results on Fields

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First set of results. Let Q be the set of all prime powers, and for $q = p^n \in Q$, p a prime, let K_q be a separably closed, non algebraically closed field of characteristic $p > 0$, and consider the difference field (K_q, σ_q), where $\sigma_q : x \mapsto x^q$. We consider the theory T consisting of all sentences true in almost all K_q, and give an axiomatisation of T. In the language obtained by adding to $\{+, \cdot, 0, 1, \sigma\}$ functions (which will be interpreted as the λ-functions of K if the characteristic is $p > 0$) and predicates for $\sigma(K)$-linear independence, we show that the expansion by definition of T is model-complete. All completions of T are simple, and various results valid in models of ACFA extend.

Second set of results. For each prime p, let $F(p)$ be the differential field $(\mathbb{F}_p(t)^s, +, \cdot, D_p)$, where t is transcendental over $\mathbb{F}_p$ and D_p is the unique derivation on $\mathbb{F}_p(t)^s$ satisfying $D_p t = 1$. We let T_D be the set of all sentences true in almost all $F(p)$. We show that T_D is bi-interpretable with a certain second-order theory of pseudo-finite fields. We also show that $T_{D\forall}$ and $T_{D\exists}$ are Σ^0_2-complete.

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