Embedding Finite Lattices Into the Lattice of Ideals of Computably Enumerable Turing Degrees

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This talk will be a survey of results concerning lattice embeddings into the lattice of ideals of computably enumerable Turing degrees, $\mathcal{I}$. The problem of embedding lattices into $\mathcal{I}$ is related to the well-known problem of embedding lattices into the computably enumerable Turing degrees, $\mathcal{R}$.

I will discuss recent joint work with Lerman in which we show that the lattice L_{20} provides an example of a finite lattice that is not embeddable into $\mathcal{I}$. The proof uses a reorganized and adapted version of Lempp and Lerman's proof that the lattice L_{20} is not embeddable into $\mathcal{R}$. We also obtain a Π_2 necessary and sufficient condition for embedding lattices into $\mathcal{I}$. The condition may be obtained by reduction to Lerman's condition for embedding ranked finite partial lattices into $\mathcal{R}$.

[1] W. C. Calhoun, The Lattice of Ideals of Recursively Enumerable Degrees, Ph.D. Thesis, University of California, Berkeley, CA (1990).

[2] S. Lempp and M. Lerman, A finite lattice that cannot be embedded into the enumerable Turing degrees, Annals of Pure and Applied Logic, Vol. 87 (1997), 167-185.

[3] M. Lerman, A necessary and sufficient condition for embedding ranked finite partial lattices into the computably enumerable degrees, Annals of Pure and Applied Logic *94* (1998) 143-180.

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