Post Problem for Restricted Reducibilities

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We consider the well known [1] problem by E.Post on construction of such sets that with respect to a given reducibility have degrees intermediate between the degrees of recursive sets and the degrees of complete sets.

Post considered reducibilities $\leq_m$, $\leq_{btt}, \leq_{tt}, \leq_T$ and solved the problem for all of them except $\leq_T$. For the latter Post constructed classes of simple and hyper-simple sets.

In [2] the following result was presented: Post's approach is applicable to any "characteristically bounded" reducibility $\leq_{TM}$ that does not coincide with $\leq_T$ on the class of recursively enumerable sets (r.e.s.). Furthermore, even the class of hyper-simple sets suffices to show that.

In this work another complexity measure for Turing reductions is considered. We call it *segment complexity* [3]. Let $A[n] = (A(i))_{i<n}$ denote the initial segment of set A. So $A(i)$ denotes the i-th element of A. The number n is the length of the segment $A[n]$. We can now say $A \leq_{MT} B$ iff there exist an index $x \in \mathbb{N}$ and a total function $h : \mathbb{N} \to \mathbb{N}$ such that the reduction $A \leq_T^x B$ takes place, $h \leq_T M$, and the initial segment of oracle $B[h(n)]$ is sufficient to compute $A[n]$ via the reduction almost for any n. For any set M relation $\leq_{MT}$ is also a reducibility with respect to decidability.

Therefore, the problem of interest is to describe such T-degrees $d_T(M)$ that Post's ideas are applicable to reducibilities $\leq_{MT}$.

As a part of our solution, we introduce *radius of recursiveness* $R_B : \mathbb{N} \to \mathbb{N}$ of any set B defined as a function that on the input n yields the length of the maximal initial segment of B computable by a program with the length no greater than n. M-radially-h-simple set is such a r.e.s. A that the radius of recursiveness $R_{\bar{A}}$ of its complement can not be majorated by any increasing total recursive in M function.

Theorem. For any set M of low Turing degree there exist M-radially-h-simple non-simple sets. All such sets are not $\leq_{MT}$-complete.

[1] Rogers H.,Jr. *Theory of Recursive Functions and Effective Computability*, McGraw-Hill, 1967.

[2] Bulitko V.K. *On Post's Problem for Reducibilities with Information Bounds*, 11^{th} International Congress of LMPS, Cracow, Poland, 1999, Volume of Abstracts, p.76.

[3] Bulitko V.K. *About Segment Complexity of Turing Reductions*, Math. Logic Quart., 45 (1999) 4, 561-571.

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