On Some Families of Thin Sets

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Let $f, g : \langle 0, 1\rangle \longrightarrow \langle, \infty)$ be continuous, $f(0) = g(0) = f(1) = g(1) = 0$. Replacing the sinus function by f the families of pseudo-Dirichlet sets $p\mathcal{D}_f$, Arbault sets $\mathcal{A}_f$ and N_{0f}-sets $\mathcal{N}_{0f}$ can be defined (see e.g. [BL]) in similar way as the trigonometric ones. For a big family of functions f, g we shall present sufficient and/or necessary conditions for the inclusions $\mathcal{N}_{0f} \subseteq \mathcal{N}_{0g}$, $\mathcal{N}_{0f} \subseteq \mathcal{A}_g$, $\mathcal{A}_f \subseteq \mathcal{A}_g$. Consequently, there exists a large hierarchy of those families.

For a family $\mathcal{F}$ of thin sets, the family $\mathrm{Prm}(\mathcal{F})$ of $\mathcal{F}$-permitted sets is defined as

$$A \in \mathrm{Prm}(\mathcal{F}) \equiv (\forall B \in \mathcal{F}) A \cup B \in \mathcal{F}.$$

We show that the estimates for several cardinal characteristics of the trigonometric families of permitted sets obtained in [KK], [RM] holds true for $\mathrm{Prm}(\mathcal{F})$, $\mathcal{F} = p\mathcal{D}_f, \mathcal{N}_{0f}, \mathcal{A}_f$, assuming that $f(x - y) \leq f(x) + f(y)$, e.g.

$$\mathrm{non(perfectly\ measure\ zero)} \leq \mathrm{non}(\mathrm{Prm}(\mathcal{F}) \leq \min\{\mathrm{non}(\mathcal{L}), \mathrm{non}(\mathcal{K})\}$$

and $\mathrm{cov}(\mathcal{F}) \leq \mathfrak{l}$ for $\mathcal{F} = p\mathcal{D}_f$, $\mathcal{N}_{0f}$, $\mathcal{A}_f$.

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[RM] Repický M., *A family of permitted trigonometric thin sets,* Proc. Amer. Math. Soc. **125** (1997), 137-144.

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