On Evidence Concerning the Continuum Hypothesis

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From results of Gödel and Cohen it follows that the continuum hypothesis (CH) is consistent with, but not provable from, ZFC, the commonly accepted axioms of set theory. Gödel had proposed that adoption of stronger axioms of infinity might settle the CH. But a result of Solovay and Levy shows that large cardinal axioms do not settle the question of CH directly. Nonetheless, W. H. Woodin has claimed that results concerning the P_{max} model, some forcing axioms, and a notion of consequence in Ω-logic provide, assuming the existence of large cardinals, evidence that the CH is false. I will present results on my attempt to extract the philosophical and methodological principles lying behind this claim and to philosophically evaluate these principles. How do these results constitute evidence that CH is false? For example, Woodin's approach to the CH seems not to be merely the usual sort of strategy of proposing a new axiom of which not-CH is a consequence and then saying why the axiom should be accepted as an axiom of set theory. What justifications might be given for the large cardinal assumptions, the forcing axioms, and other assumptions used to make Woodin's claim? For example, the notion of consequence in Ω-logic involves a restriction to a certain class of models. What justifies this restriction?

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