# Poly-Regular Ordered Abelian Groups

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An ordered abelian group is called *regular* if for any positive integer $n$ every interval of cardinality at least $n$ contains an $n$-divisible element. A. Robinson and E. Zakon proved that a group is regular iff it is elementarily equivalent to an archimedian group. We call an ordered abelian group $G$ *poly-regular* if there are $n < \omega$ and a series of convex subgroups

$$0 = G_0 < G_1 < \ldots < G_n = G$$

such that $G_{i+1}/G_i$ is regular for all $i < n$. We call such a series poly-regular. The minimal $n$ with this property is called the *poly-regular rank* (or, for short, just the *rank*) of $G$. So the regular groups are exactly the poly-regular groups of rank 1. Clearly, any lexicographically ordered direct sum of finitely many regular groups is a poly-regular group.

**Theorem 1** *Any poly-regular group $G$ has a unique poly-regular series*

$$0 = G_0 < G_1 < \ldots < G_n = G$$

*such that $G_{i+1}/G_i$ is not divisible for $1 \leq i < n$. The number $n$ is equal to the poly-regular rank of $G$. All members of the series are 0-definable subgroups.*

**Corollary 2** *Any group elementarily equivalent to a poly-regular group is poly-regular of same rank. The class of poly-regular groups of rank at most $n$ is first order axiomatizable.*

**Theorem 3** *A group is poly-regular iff it is elementarily equivalent to a lexicographically ordered direct sum of finitely many archimedian groups. More precisely, for an ordered abelian group $G$ and a positive integer $n$ the following are eqivalent:*

(1) *$G$ is poly-regular of rank $n$,*

(2) *$G$ has exactly $n-1$ definable proper convex subgroups,*

(3) *$G$ is elementarily equivalent to the lexicographically ordered direct sum of archimedian groups $A_1, \ldots, A_n$, where $A_1, \ldots, A_{n-1}$ are not divisible.*

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