Applications of Quantifier Elimination in Banach Spaces

Stefano Baratella & Siu-Ah Ng

Following an approach to quantifier elimination pursued in an abstract setting by Keisler [K], in [BN] we introduced a notion of quantifier elimination (briefly: QE) for structures based on Banach spaces. Such a notion can be proved equivalent to Henson's quantifier elimination in sufficiently saturated nonstandard hulls. QE refers to a class of infinitary expressions, called by Keisler *neocompact formulas*, extending the class of *positive bounded formulas* introduced in [H].

First we present some properties equivalent to QE, then we discuss consequences of QE (or weaker properties like *density* or *back-and-forth*) on the geometry of a Banach space.

Eventually, we point out relationship between the abovementioned properties and fixed point theorems in reflexive nonstandard hulls of Banach spaces.

[BN] S. Baratella and S.-A. Ng, **Neocompact quantifier elimination in structures based on Banach spaces**, to appear in *Annals of Pure and Applied Logic.*
[H] C.W. Henson, **Nonstandard hulls of Banach spaces**, *Israel J. Math.* **25** (1976), pp. 108-144.
[K] H.J. Keisler, **Quantifier elimination for neocompact sets**, *Journal of Symbolic Logic* **63**, no. 4 (1998), pp. 1442-1472.

Addresses
Stefano Baratella: Dipartimento di Matematica, via Sommarive 14, 38050 Povo TN - Italia
E-mail: `baratell@science.unitn.it`

Siu-Ah Ng: University of Natal, Pietermaritzburg, 3209 South Africa
E-mail: `ngs@math.unp.ac.za`