Universal Set Theory and Strong Modal Reflection Principles

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I consider infinitary modal logic $L^{\Diamond}_{\infty}$ as a foundational language for set theory. The Barwise-Moss semantics for modal logic (see [4]) is given by considering sets and classes as Kripke structures, with converse-membership as the accessibility relation. I use this semantics to *axiomatize and model* a non-classical, non-wellfounded universe of sets, containing a "universal" set U and other "large" sets. This universe can be seen as the largest extension of Peter Aczel's non-wellfounded universe which still satisfies Aczel's Strong Extensionality principle. My central axiom $SAFA$ ("Super-Antifoundation Axiom") is a strengthening of Aczel's AFA, stated as a *strong Reflection Principle* $L^{\Diamond}_{\infty}$. My consistency proof for this set theory uses a generalization of the standard "canonical model" construction for modal logic. The resulting model is isomorphic to one of the models constructed by researchers on "topological set theory" ([5], [6] etc). I use this set theory to give non-classical solutions to classical set-theoretical paradoxes, to prove fixed-point theorems relating recursion and corecursion, to formalize a "self-referential" model theory and a "reflexive" category theory, to provide natural solutions for domain equations, and to justify some well-known "large cardinal" axioms.

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