Determinacy and Weakly-Ramsey Sets in Banach Spaces

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We give a sufficient condition for a set of block subspaces in an infinite-dimensional Banach space to be weakly-Ramsey ([2]). Using this condition we prove that in the Levy-collapse of a Mahlo cardinal over L, every projective set is weakly-Ramsey. This, together with a construction of W. H. Woodin ([4]), is used to show that the Axiom of Projective Determinacy implies that every projective set is weakly-Ramsey. For Hereditarily Indecomposable spaces we show that the Axiom of Determinacy plus the Axiom of Dependent Choices imply that every set is weakly-Ramsey. These results are the generalizations to the class of projective sets of some theorems from W. T. Gowers [2], [3], and our paper [1].

[1] J. Bagaria and J. Lpez-Abad, Weakly-Ramsey sets in Banach spaces. Submitted.

[2] W. T. Gowers, Analytic sets and games in Banach spaces. Preprint IHES M/94/42.

[3] W. T. Gowers, An infinite Ramsey theorem and some Banach-space dichotomies. Preprint.

[4] W. H. Woodin, On the consistency strength of projective uniformization, in *Proceedings of Herbrand Symposium. Logic Colloquium '81*, North-Holland, 1982.

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