On Neat Reducts of Algebras of Logics

Tarek Sayed Ahmed

SC_α, CA_α, QA_α and QEA_α stand for the classes of Pinter's substituton algebras, Tarski's cylindric algebras, Halmos' quasipolyadic algebras, and quasipolyadic equality algebras of dimension α, respectively. Generalizing a result of Nemeti on cylindric algebra, we show that for $K \in \{SC, CA, QA, QEA\}$, and ordinals $\alpha < \beta$, the class $Nr_\alpha K_\beta$, of α-dimensional neat reducts of β-dimensional K-algebras, though closed under taking homomorphic images and products, is not closed under forming subalgebras (i.e.. is not a variety), iff $\alpha > 1$.

From this it easily follows that for $1 < \alpha < 0$, the operation of forming α-neat reducts of algebras in K_β do not commute with forming subalgebras, a notion to be made precise.

We give a contrasting result concerning Halmos' polyadic algebras (with and without equality). For such algebras, we show that the class of infinite dimensional neat reducts forms a variety.

We comment on the status of the property of neat reducts commuting with forming subalgebras for various reducts of polyadic algebras that are also expansions of cylindric like algebras. We try to draw a borderline between reducts that have this property and reducts that do not.

Following research initiated by Pigozzi, we also emphasize on the strong tie that links the (apparently non-related) property of neat reducts commuting with forming subalgebras, to proving amalgamation results in CA-like algebras of relations. We show that (like amalgamation) neat reducts commuting with forming subalgebras, is another algebraic expression of definability, and hence is also strongly related to the well known metalogical properties of Craig, Beth and Robinson in the corresponding logics.

Address: Department of Mathematics, Faculty of Science, Cairo University, Egypt.